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Interacting magnons and critical behavior of bosons

Part A: Magnon-phonon interactions and non-equilibrium dynamics of magnons in Yttrium-Iron Garnet

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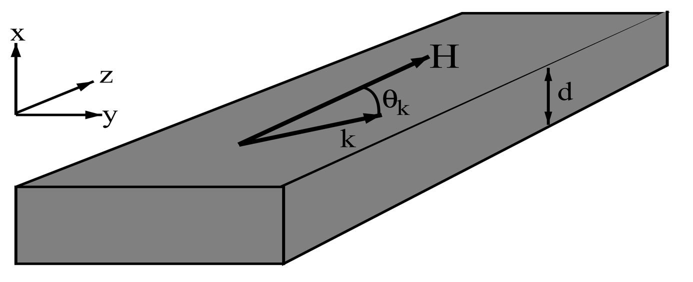
Magnons in thin YIG films

1 Spin Hamiltonian

- Ferromagnetic quantum Heisenberg model for lowest magnon band:

$$H = -\mu H \sum_i S_i^z - \frac{1}{2}\sum_{ij} J_{ij}\mathbf{S}_i\cdot\mathbf{S}_j - \frac{1}{2}\sum_{i\neq j}\frac{\mu^2}{|\mathbf{r}_{ij}|^3}\left[3(\mathbf{S}_i\cdot\hat{\mathbf{r}}_{ij})(\mathbf{S}_j\cdot\hat{\mathbf{r}}_{ij}) - \mathbf{S}_i\cdot\mathbf{S}_j\right]$$

- Effective spin: $S \approx 14.2$



Geometry of a thin, tangentially magnetized YIG stripe with thickness d. The lowest magnon band is obtained by only considering magnons with in-plane wave vectors.

2 Spin wave expansion

- Holstein-Primakoff bosonization:

$$S_i^z = S - b_i^\dagger b_i$$
$$S_i^+ = \left(S_i^-\right)^\dagger = \sqrt{2S}\sqrt{1-\frac{b_i^\dagger b_i}{2S}}\, b_i$$
$$S_i^\pm = S_i^x \pm i S_i^y$$

- $1/S$ expansion → interacting Bose gas
- Dipolar interactions: Bogoliubov transformation $b_k \to \beta_k$
- Magnon Hamiltonian:

$$H = E_0 + \sum_k E_k \beta_k^\dagger \beta_k + \mathcal{O}(S^0)$$

- Magnon dispersion:

$$E_k = [\mu H + \rho_s k^2 + \Delta(1-f_k)\sin^2\theta_k]^{1/2} \times [\mu H + \rho_s k^2 + \Delta f_k]^{1/2}$$

Magnetoelastic interaction

1 Phenomenological theory

- No microscopic theory available to describe the dominant, relativistic source of spin-lattice interactions in collinear magnets
- Phenomenological classical magnetoelastic energy:

$$E_{\rm me} = \frac{n}{M_s^2}\int d^3r \sum_{\alpha\beta} B_{\alpha\beta} M_\alpha(r) M_\beta(r) X_{\alpha\beta}(r)$$

  - Symmetric strain tensor:

$$X_{\alpha\beta}(r) = \frac{1}{2}\left[\frac{\partial X_\alpha(r)}{\partial r_\beta} + \frac{\partial X_\beta(r)}{\partial r_\alpha}\right]$$

  - $X(r)$: Elastic displacement field
  - $M(r)$: Macroscopic magnetization
  - Coupling tensor $B_{\alpha\beta}$ known from experiment

2 Quantization

- Magnons: $M(R_i) \to \mu n S_i$
  - Holstein-Primakoff bosonization
  - $1/S$ expansion
  - Bogoliubov transformation
- Phonons:

$$X(k) = \int d^3r e^{-ik\cdot r} X(r) \to V X_k/\sqrt{N}$$

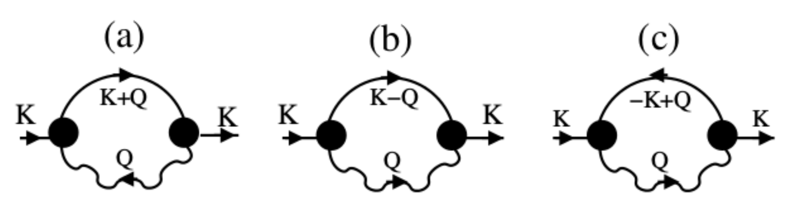
$$X_k = \sum_\lambda X_{k\lambda} e_{k\lambda} = \sum_\lambda \frac{a_{k\lambda} + a_{-k\lambda}^\dagger}{\sqrt{2m\omega_{k\lambda}}} e_{k\lambda}$$

  - $a_{k\lambda}^\dagger$: Creates acoustic phonon with energy $\omega_{k\lambda} = c_\lambda k$ and polarization vector $e_{k\lambda}$
  - Effective ionic mass m and sound velocities c_λ known from experiment

Magnon damping

1 Theory

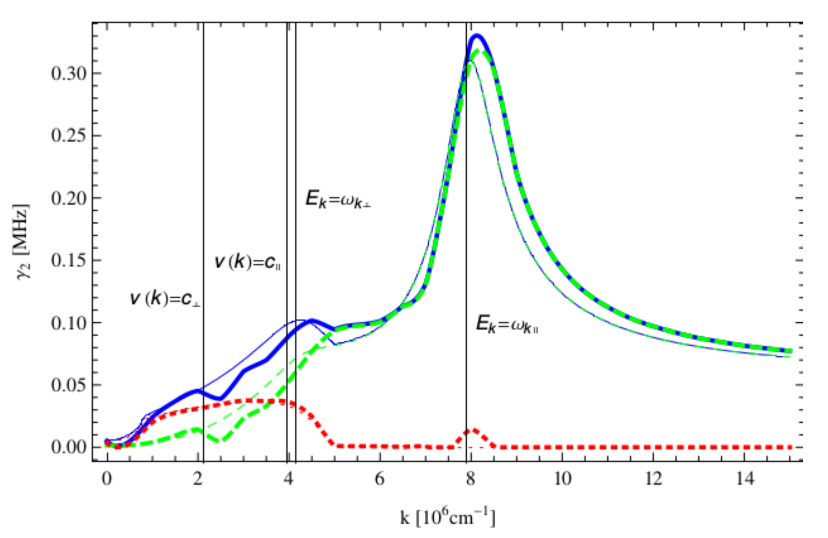
- Diagrams determining the leading order (S^{-2}) magnon damping:



(a) Cherenkov absorption process

(b) Cherenkov emission process

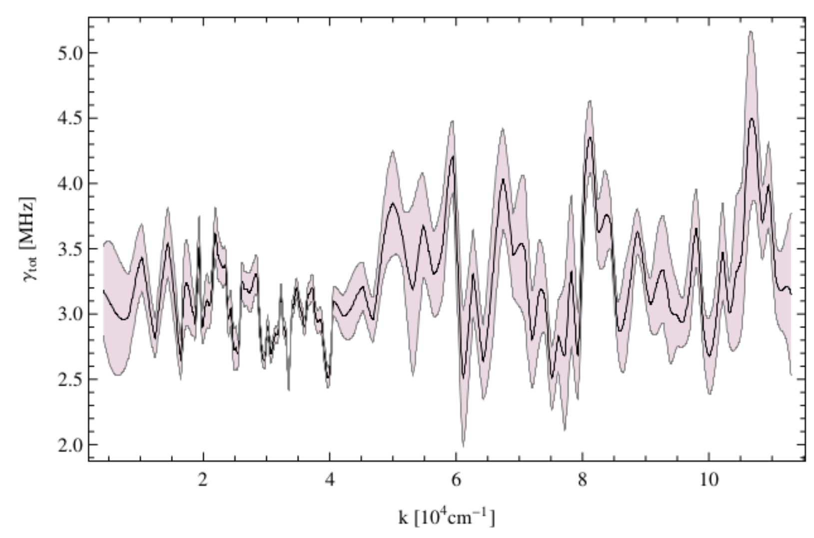
(c) Confluent process



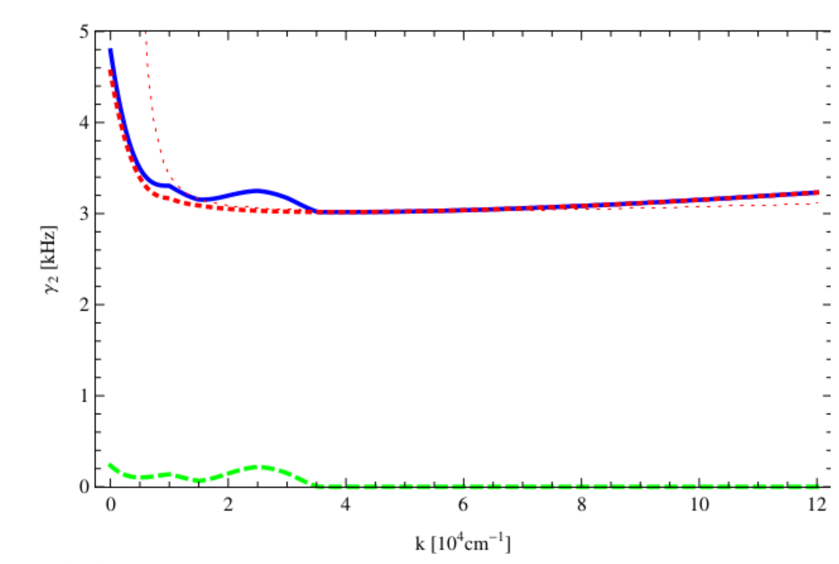
Calculated magnon-phonon damping rate in the exchange momentum regime, for $d = 6.7\ \mu$m, $H = 1710$ Oe, $T = 300$ K, and k parallel to the external field. Blue lines correspond to the total damping rate, green and red to Cherenkov and confluent processes respectively. Thin lines are approximations neglecting the dipolar interactions.

2 Experiment

- Time- and wave-vector-resolved Brillouin light spectroscopy **A7**
- Free relaxation after inital magnon injection by parallel pumping at ferromagnetic resonance



Total magnon damping measured at room temperature for $d = 6.7\ \mu$m, $H = 1710$ Oe, and k parallel to the external field.



Calculated magnon-phonon damping rate in the dipolar momentum regime, for $d = 6.7\ \mu$m, $H = 1710$ Oe, $T = 300$ K, and k parallel to the external field. Blue lines correspond to the total damping rate, green and red to Cherenkov and confluent processes respectively. The thin red line is an analytical approximation for the confluent contribution.

Effective magnon action

- Magnetoelastic interactions linear in phonon operators → phonon degrees of freedom can be integrated out
- Effective magnon action in Matsubara formalism with $K = (k, i\omega)$:

$$S[\bar\beta,\beta] = -\frac{1}{T}\sum_K\left\{[i\omega - E_k - \Sigma_1(K)]\bar\beta_K\beta_K - \frac{1}{2}\left[\Pi_1(K)\beta_{-K}\beta_K + \Pi_1^*(K)\bar\beta_K\bar\beta_{-K}\right]\right\} + S_{\rm int}[\bar\beta,\beta]$$

- Σ_1, Π_1: Diagonal and off-diagonal self-energies of order S^{-1}
- $S_{\rm int}$ contains cubic interactions $\propto S^{-3/2}$ and quartic interactions $\propto S^{-2}$

Magnetoelastic modes

1 Dispersion

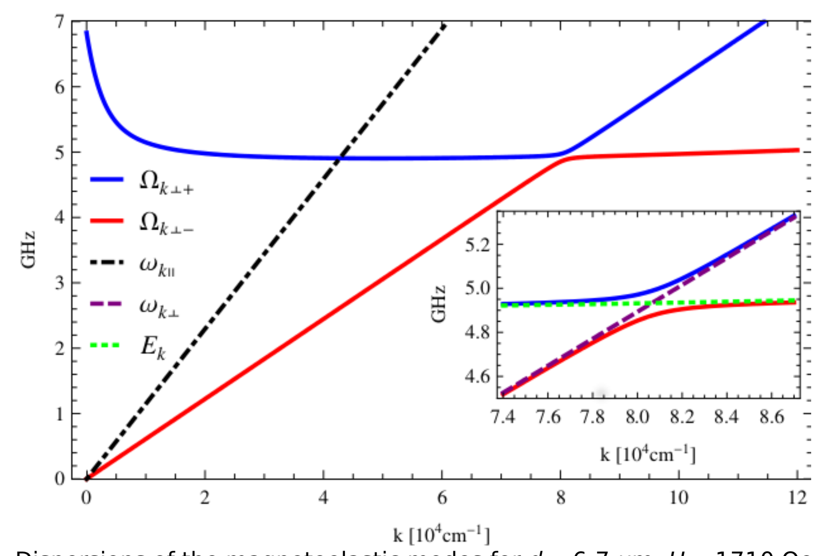
- Off-diagonal self-energy Π generated → matrix Green function $\hat{G}$
- Magnetoelastic modes are obtained as roots of

$$\det\left[\hat{G}(k,\omega+i0^+)\right]^{-1} = 0$$

- Solution to order S^{-1} for a given phonon branch:

$$\Omega^2_{k\lambda\pm} = \frac{\omega^2_{k\lambda} + E_k^2}{2} \pm \sqrt{\frac{\left(\omega^2_{k\lambda} - E_k^2\right)^2}{4} + \Delta^4_{k\lambda}}$$

$$\Delta^4_{k\lambda} = 2\frac{E_k}{m}|\Gamma^\beta_{k\lambda}|^2$$



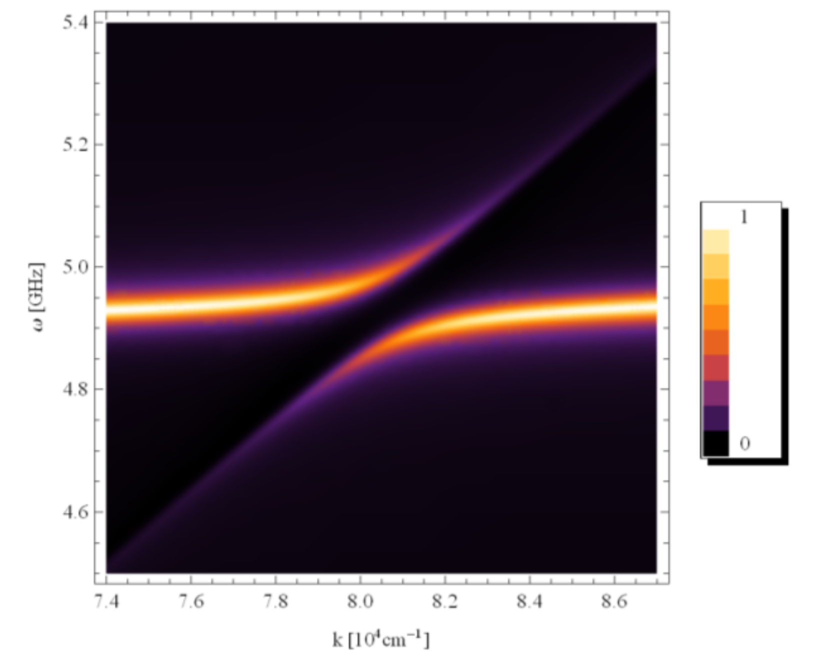
Dispersions of the magnetoelastic modes for $d = 6.7\ \mu$m, $H = 1710$ Oe, and k parallel to the external field.

2 Spin structure factor

- Transverse spin structure factor, same approximation:

$$S_\perp(k,\omega) = \frac{S Z_{k\lambda}(\omega)}{1-e^{-\omega/T}}\sum_{s=\pm} s \times [\delta(\omega - \Omega_{k\lambda s}) + \delta(\omega + \Omega_{k\lambda s})]$$

  - proportional to measured Brillouin light scattering intensity



Intensity plot of the transverse spin structure factor close to the hybridization area, for $d = 6.7\ \mu$m, $H = 1710$ Oe, $T = 300$ K, and k parallel to the external field.

Reference

A. Rückriegel, P. Kopietz, D. A. Bozhko, A. A. Serga, and B. Hillebrands, Phys. Rev. B **89**, 184413 (2014).

RESARCH PLAN

- Solve non-Markovian dissipative Landau-Lifshitz-Gilbert equation:

$$\partial_t \mathbf{S}_i(t) = \mathbf{S}_i(t)\times\left[\mathbf{H}_i(t) + \mathbf{h}_i(t) + \sum_j \hat{\mathbf{J}}_{ij}\cdot\mathbf{S}_j(t)\right] - \mathbf{S}_i(t)\times\int_0^t dt'\,\hat{\mathbf{G}}_{ij}(t,t')\cdot\partial_{t'}\mathbf{S}_j(t')$$

$$G^{\alpha\beta}_{ij}(t,t') = \frac{1}{S^4}\sum_{\mu\nu} B_{\alpha\mu}B_{\nu\beta}S_i^\mu(t)S_j^\nu(t') \times \frac{1}{N}\sum_{\mathbf{k}\lambda} e^{i\mathbf{k}\cdot(\mathbf{R}_i-\mathbf{R}_j)}\frac{(\mathbf{k}_{\alpha\mu}\cdot\mathbf{e}_{\mathbf{k}\lambda})(\mathbf{k}_{\nu\beta}\cdot\mathbf{e}_{-\mathbf{k}\lambda})}{m\omega^2_{\mathbf{k}\lambda}}\cos\left[\omega_{\mathbf{k}\lambda}(t-t')\right]$$

$$\langle h_i^\alpha(t) h_j^\beta(t')\rangle = T G^{\alpha\beta}_{ij}(t,t')$$

- Stochastic properties of random field are known from phonon dynmaics
- analytical understanding in Markovian limit: classical Langevin equation
  - effective field theory
  - non-equilibrium Functional RG
- connects to projects **A7** **A3** **B1**

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